1. C	11. A	21. D
2. D	12. C	22. E
3. D	13. C	23. B
4. B	14. B	24. C
5. D	15. B	25. A
6. DorE	16. A	26. C
7. D	17. A	27. A
8. C	18. C	28. C
9. B	19. B	29. D
10. C	20. D	30. C

1. C $3 \cdot 68 - 2 \cdot 7 = 204 - 14 = 190.$

2. D The vector from the prey to the point on the plane closest to the plane must be normal to the plane. Since the normal vector to the plane is $\langle 3,2,2 \rangle$, we can set up the equation $\langle a, b, c \rangle = t\langle 3,2,2 \rangle + \langle 3,5,7 \rangle$. This produces 3 equations (1 for each component of the vector) but we have 4 variables. The last equation is that our point lies on the plane, so $3a + 2b + 2c = 16$. Solving for a, b, and c in terms of t and plugging into the last equation, we have that $3(3t + 3) + 2(2t + 5) + 2(2t + 7) = 16 \rightarrow 9t + 9 + 4t + 10 + 4t + 14 = 17t + 33 = 16 \rightarrow 17t = -17 \rightarrow t = -1$. Note that we don't need to solve for a a, b, and c separately because we are asked for $a + b + c = 3 + 5 + 7 + 3t + 2t + 2t = 15 + 7t = 15 - 7 = 8$.

3. D The easiest way to approach this is to test the answer choices using the definition of an eigenvector ($M\vec{v} = \lambda\vec{v}$). The only answer choice that satisfies the definition is $\begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}$ because $\begin{bmatrix} 3 & -2 & 4 \\ 5 & 12 & 7 \\ 7 & -2 & -3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix} = \begin{bmatrix} -12 \\ -6 \\ 24 \end{bmatrix} = -6 \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}$

4. B The reduced row echelon form of any square matrix with linear independent columns is the identity matrix. Therefore, the reduced row echelon form is the 2 by 2 identity matrix, in which the sum of the elements is 2.

5. D The cofactor matrix, which is the matrix of minors multiplied by $(-1)^{i+j}$, where the minor has been created by taking the determinant of the submatrix created by deleting the ith row and jth column. $(-1)^3\big(-4 \cdot 6 - (-2) \cdot (-8)\big) = -(-24 - 16) = 40$.

6. E An elementary matrix is one that is one row operation away from an identity matrix.

I is false. As a counterexample, consider $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

II is true or false. The counterexample is the "trivial" case of $M = [1]$ but views differ.

III is true. Each type of row operation (row switching, row scaling, and row addition) lead to the respective changes in the target matrix that are equivalent to multiplying by an elementary matrix. For example, an elementary matrix formed by switching the last 2 rows of the 3 by 3 identity matrix would have the effect of switching the last 2 rows of any target matrix.

7. D An idempotent matrix M satisfies $M^2 = M$. Answer choice A is nilpotent, Answer choice B is periodic with period 3 ($M^4 = M$), but choice D is the only idempotent matrix: $\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}^2 = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$.

8. C To make the computation cleaner, we introduce a parameter $x = \sqrt{17}$. Then, $\langle x, 2x, 3x \rangle \times \langle 1-x, 2-x, 3-x \rangle = x \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 1 & 2 & 3 \\ 1-x & 2-x & 3-x \end{vmatrix} =$ $x \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ -x & -x & -x \end{vmatrix} = -x^2 \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = \langle x^2, -2x^2, x^2 \rangle$.
Overall, $\|\langle \sqrt{17}, 2\sqrt{17}, 3\sqrt{17} \rangle \times \langle 1-\sqrt{17}, 2-\sqrt{17}, 3-\sqrt{17} \rangle\| = \sqrt{6}\sqrt{17}^2 = 17\sqrt{6}$.

9. B As hinted at in the description, left-handed systems have the same fundamental properties as right-handed systems and are rejected only due to convention. Therefore, we can consider each of the properties in the context of the normal unit vectors and product definitions.
Cross products are anticommutative, not commutative, so I is false.
Cross products are not associative so II is false.
Cross products do distribute over addition (e.g., $\boldsymbol{i} \times (\boldsymbol{j} + \boldsymbol{k}) = \boldsymbol{i} \times \boldsymbol{j} + \boldsymbol{i} \times \boldsymbol{k}$ so III is true.
Cross products do scale over constant factors (e.g., $2\boldsymbol{i} \times \boldsymbol{j} = 2(\boldsymbol{i} \times \boldsymbol{j})$) so IV is true.
Overall, 2 of the statements are true (III and IV)

10. C It greatly speeds up the computation to note that after any even number of steps, there are 4 vertices at which the ant can be (and the other 4 are achievable with an odd number of steps). Formally, the adjacency list represents a bipartite graph. Now, we consider the transition matrix corresponding to taking 2 steps. The probability of ending back at the starting point is $3 * \frac{1}{3} * \frac{1}{3} = \frac{1}{3}$. The probability of going elsewhere is $1 - \frac{1}{3} = \frac{2}{3}$. Each other vertices is symmetric, so the individual probability is $\frac{\frac{2}{3}}{3} = \frac{2}{9}$. The transition matrix for these 2 states (the current vertex vs everywhere else) is $\begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{9} & \frac{7}{9} \end{bmatrix}$. After 4 minutes, the matrix is $\begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{9} & \frac{7}{9} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{9} & \frac{7}{9} \end{bmatrix} = \begin{bmatrix} \frac{7}{27} & \frac{20}{27} \\ \frac{20}{81} & \frac{61}{81} \end{bmatrix}$. The top left element in the matrix multiplication $\begin{bmatrix} \frac{7}{27} & \frac{20}{27} \\ \frac{20}{81} & \frac{61}{81} \end{bmatrix}$, which represents the probability of being at the original vertex after 4 minutes.

11. A I is true; it is essentially the definition of linear algebra, and why it is called "linear".
II is false because even though linear transformations are usually applied to vectors, a transformation can be applied to another transformation to produce a compound transformation.
III is false because matrices can have arbitrarily many rows/columns which represent increasing dimensionality.

12. C This problem requires us to diagonalize the matrix. The first step in doing this is to find the eigenvalues: $\begin{vmatrix} -2-\lambda & 4 \\ -3 & 5-\lambda \end{vmatrix} = (\lambda-5)(\lambda+2)+12 = \lambda^2-3\lambda+2 = (\lambda-1)(\lambda-2)$. Therefore, the eigenvalues are 1 and 2. The eigenvector corresponding to 1 satisfies $\begin{bmatrix} -2 & 4 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \to 4b-2a = a \to a = \frac{4}{3}b$, so the eigenvector is $\begin{bmatrix} \frac{4}{3} \\ 1 \end{bmatrix}$. For the eigenvalue of 2, we have $\begin{bmatrix} -2 & 4 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2a \\ 2b \end{bmatrix} \to 4b-2a = 2a \to a = b$. So, the eigenvector is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.
For the question given, we can compute $SD^{100}\,S^{-1}$ where $S = \begin{bmatrix} \frac{4}{3} & 1 \\ 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$. $D^{100} = \begin{bmatrix} 1 & 0 \\ 0 & 2^{100} \end{bmatrix}$, $S^{-1} = \begin{bmatrix} 3 & -3 \\ -3 & 4 \end{bmatrix}$. Doing the multiplication, $\begin{bmatrix} \frac{4}{3} & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 2^{100} \end{bmatrix} \cdot \begin{bmatrix} 3 & -3 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} \frac{4}{3} & 2^{100} \\ 1 & 2^{100} \end{bmatrix} \cdot \begin{bmatrix} 3 & -3 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 4-3\cdot 2^{100} & -4+4\cdot 2^{100} \\ 3-3\cdot 2^{100} & -3+4\cdot 2^{100} \end{bmatrix}$.

13. C The direction vector along the first line is $\langle 6,3,2\rangle$ and the direction vector along the second line is $\langle 2,3,6\rangle$. Taking the cross product, $\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 6 & 3 & 2 \\ 2 & 3 & 6 \end{vmatrix} = 12i - 32j + 12k$. Therefore, the plane $12x - 32y + 12z = c$ contains both direction vectors. Fitting the first line on the plane, the point $(-1,-1,-1)$ is on the line so $c = 32-12-12 = 8$. Dividing by 4, the plane becomes $3x-8y+3z = 2$. We can now apply the point to plane formula using an arbitrary point on the second line. The point $(1,2,6)$ is on the second line, so our formula becomes $\frac{|1\cdot 3-2\cdot 8+3\cdot 6-2|}{\sqrt{3^2+8^2+3^2}} = \frac{3}{\sqrt{82}}$

14. B A matrix is nilpotent if and only if it raising it to a finite power results in the zero matrix. First, note that in the case of a binary matrix, all of the elements of the main diagonal must be 0. Otherwise, any power of the matrix will have a nonzero element in that position. Now note that the matrix $M = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is nilpotent since $M^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $M^4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. We claim that there cannot be more than 3 nonzero elements in a binary nilpotent matrix. To see why, note that with 4 or more elements, since we must avoid the main diagonal there will be a pair of nonzero elements symmetric across the main diagonal. This results in a matrix that is not nilpotent because the two nonzero elements symmetric across the main diagonal will always be present in powers of the matrix due to symmetry. For example, $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

15. B The determinant of the matrix is $6(3+2k) - 2k\cdot k^2 = -2k^3 + 12k + 18$. Dividing by -2, $k^3 - 6k - 9 = 0$, which has $k = 3$ as its only real root.

16. A For this system, State 3 is a sink; once the system enters State 3 it cannot leave. Because States 1 and 2 are connected with State 4, and State 4 is connected to State

3, after a sufficient period, essentially the entire probability of the system will be concentrated in State 3. Therefore, the probability of being in State 1 is 0.

17. A The magnitude of the Coriolis force is $2m \cdot 7.3 \cdot 10^{-5} \cdot ||v|| \cdot \sin\theta$. In this case, $\theta = 60$ degrees because the rotation axis is directed north, and the wind speed is $90 - 30 = 60$ degrees separated from that.

Plugging in the given values, the magnitude is $2 \cdot 4000 \cdot 7.3 \cdot 10^{-5} \cdot 30 \cdot \frac{\sqrt{3}}{2} = 8 \cdot 7.3 \cdot \frac{15\sqrt{3}}{100} = 6 \cdot \frac{7.3\sqrt{3}}{5} = \frac{219\sqrt{3}}{25}$

18. C $1 * 4 + 2 * 5 + 3 * 6 = 4 + 10 + 18 = 32$.

19. B First off, the maximum value of $\sin x + \cos x$ occurs at $x = \frac{\pi}{4}$, so the maximum is $\sqrt{2}$. The minimum is the same as the minimum of $\sin x + \cos x$ where $0 \le x \le \frac{\pi}{2}$. This occurs at the endpoints, for a minimum of 1.

The characteristic polynomial of the matrix is $(\lambda - 1)(\lambda - \sqrt{2})$, so $a + c = 1 + \sqrt{2}$. The determinant is the product of the eigenvalues so $ac - 1 = \sqrt{2} \rightarrow ac = 1 + \sqrt{2}$.

$a^2 + c^2 = (a + c)^2 - 2ac = \left(1 + \sqrt{2}\right)^2 - 2\left(1 + \sqrt{2}\right) = \left(1 + \sqrt{2} - 2\right)\left(1 + \sqrt{2}\right) = \left(\sqrt{2} - 1\right)\left(\sqrt{2} + 1\right) = 2 - 1 = 1$.

20. D The equation for the top right entry is $12c = 36$, so $c = 3$. For the bottom left element, it is $4a = 12$, so $a = 3$. Therefore, $ac = 3 * 3 = 9$. From the equation for the bottom right element, $ac + bd = 23$, so $bd = 23 - ac = 23 - 9 = 14$. Finally, our answer is $abcd = (ac) \cdot (bd) = 9 \cdot 14 = 126$.

21. D $\begin{bmatrix} a & 0 \\ b & c \end{bmatrix} \cdot \begin{bmatrix} d & e \\ 0 & f \end{bmatrix} = \begin{bmatrix} ad & ae \\ bd & be + cf \end{bmatrix}$. If $ad = 0$ then either $a = 0$ or $d = 0$, which would make either ae or bd also 0. Thus, M cannot be decomposed. Observing $N = IN$ gives an LU decomposition for N.

22. E In an LU decomposition, the determinant is easy to compute given the decomposition. Specifically, $\det\left(\begin{bmatrix} a & 0 \\ b & c \end{bmatrix} \cdot \begin{bmatrix} d & e \\ 0 & f \end{bmatrix}\right) = \det\left(\begin{bmatrix} a & 0 \\ b & c \end{bmatrix}\right) \cdot \det\left(\begin{bmatrix} d & e \\ 0 & f \end{bmatrix}\right) = ac \cdot df = acdf$. Since the main diagonal elements are integers greater than 2, the minimum value of the determinant is $2^4 = 16$. Since none of the given matrices have determinants of at least 16, none of them are expressible in the form requested.

23. B This question is essentially asking how to calculate the intersection of two planes but in reverse. One way to approach the question is to note that since the line is contained in both planes, it must be normal to the normal vectors of both the planes. Therefore, the direction vector of the line is the cross product of the two normal vectors. We can compute the cross products with $\overrightarrow{n_1} = \langle 5,6,7 \rangle$ one at a time using the answer choices. The only answer that works is $\overrightarrow{n_2} = \langle -1,2,3 \rangle$.

$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 5 & 6 & 7 \\ -1 & 2 & 3 \end{vmatrix} = 4i - 22j + 16k$ which scales to $\langle 2, -11, 8 \rangle$.

24. C By symmetry, $\vec{o} = \lambda\langle 1,1 \rangle$ where λ is some arbitrary constant.

$$\cos\theta = \frac{(\vec{o} \cdot \vec{n})}{||\vec{o}||||\vec{n}||} = \frac{7\lambda + \lambda}{\sqrt{\lambda^2 + \lambda^2} \cdot \sqrt{7^2 + 1^2}} = \frac{8\lambda}{\lambda\sqrt{2} \cdot 5\sqrt{2}} = \frac{8}{10} = \frac{4}{5}$$

25. A To be compatible, the number of columns of the matrix on the left must be equal to the number of rows of the matrix on the right. There are only $4 \cdot 3 = 12$ possibilities, so they can be enumerated by casework on the left matrix. The possible multiplications are $A \cdot C, B \cdot D, C \cdot D$, and $D \cdot A$, for a total of 4.

26. C This can be done with a rotation matrix. The angle between the lines is $\theta = \frac{\pi}{3} + \frac{\pi}{4} = \frac{7\pi}{12}$ (note that it is also possible to use $\theta = -\frac{5\pi}{12}$ due to the symmetry of the line about the origin). Finally, our rotation matrix is $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}-\sqrt{6}}{4} & \frac{-\sqrt{6}-\sqrt{2}}{4} \\ \frac{\sqrt{6}+\sqrt{2}}{4} & \frac{\sqrt{2}-\sqrt{6}}{4} \end{bmatrix}$.

27. A The key bit of knowledge to know for this question is that shoelace produces a raw value that is positive when done using points going counterclockwise and this it is negative when points are going clockwise. A counterclockwise path would always be to the right of interior points, thereby keeping $(3, 7)$ to the left of it. Therefore, we count the number of positive values; there are 4.

28. C The characteristic polynomial of the matrix is the following determinant: $\begin{vmatrix} 3-\lambda & 4 & 2 \\ 5 & 3-\lambda & 7 \\ 1 & 0 & 4-\lambda \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 3-\lambda & 7 \end{vmatrix} + (4-\lambda) \cdot \begin{vmatrix} 3-\lambda & 4 \\ 5 & 3-\lambda \end{vmatrix} = 28 + 2 \cdot (\lambda - 3) + (4-\lambda) \cdot [(\lambda-3)^2 - 20] = -(\lambda^2 - 6\,\lambda - 11) \cdot (\lambda - 4) + 2\lambda + 22 = -(\lambda^3 - 10\lambda^2 + 11\lambda + 22)$. The roots of this equation are the eigenvalues, so the sum of the eigenvalues taken two at a time is $\frac{c}{a} = 11$.

29. D According to the Parallelogram Law for vector addition, $\|v+w\|^2 + \|v-w\|^2 = 2\|v\|^2 + 2\|w\|^2$. Plugging in the givens, we have that $\|v-w\|^2 = 2(2)^2 + 2(1)^2 - \left(2\sqrt{2}\right)^2 = 8 + 2 - 8 = 2$. Therefore, $\|v-w\| = \sqrt{2}$.

30. C The trace is the sum of the elements along the main diagonal and for the identity matrix all these elements are 1, so $10 \cdot 1 = 10$.