1. C
2. A
3. B
4. E
5. C
6. B
7. B
8. B
9. C
10. D
11. D
12. D
13. B
14. E
15. A
16. D
17. A
18. C
19. D
20. E
21. A
22. B
23. E
24. C
25. A
26. E
27. A
28. D
29. B
30. A

1. C Note that if n is even, $(-n)i^{-n} + ni^n = 0$, if n is odd $(-n)i^{-n} + ni^n = 2ni^n$
So, the sum is
$$2(9i - 7i + 5i - 3i + i) = 10i$$
2. A $|(2+i)^{10}(3+4i)| = |2+i|^{10}|3+4i| = \sqrt{5}^{10} \cdot 5 = 5^5 \cdot 5 = 3125 * 5 = 15625$
3. B $\arg(-4+i) = \pi - \tan^{-1}\frac{1}{4}$, $\arg(-1-4i) = -\frac{\pi}{2} - \tan^{-1}\frac{1}{4}$. Thus the positive difference between the two arguments is $\frac{3\pi}{2}$.
4. E Using the fact that $\cosh(\theta) = \frac{e^\theta + e^{-\theta}}{2}$, $2\cosh(\theta) = e^\theta + e^{-\theta}$. By inspection, $x = e^\theta$ (or $e^{-\theta}$, but the answer remains the same). $x^2 + \frac{1}{x^2} = e^{2\theta} + e^{-2\theta} = 2 * \frac{e^{2\theta}+e^{-2\theta}}{2} = 2\cosh(2\theta)$.
5. C Let $x = 5z$. Then, $x^5 = (5z)^5 = 5^5 * z^5 = 3125\,z^5 = 3125$. So, $z^5 = 1$. We will solve for the requested product with this scaled version of the problem. Then, since we are computing the product of 4 distances, and each distance was scaled down by a factor of 5, we will multiply by $5^4 = 625$. $z^5 - 1 = 0$ holds. $z^5 - 1 = (z-1)(z^4 + z^3 + z^2 + z + 1) = 0$. WLOG let M lie on the real axis, so M represents the real root $z = 1$. For the other 4 roots, it is true that $z^4 + z^3 + z^2 + z + 1 = 0$. Letting the others roots be r_1, r_2, r_3, and r_4, this means $(z-r_1)(z-r_2)(z-r_3)(z-r_4) = z^4 + z^3 + z^2 + z + 1$. The product of the distances from z = 1 to the other roots is $|(1-r_1)(1-r_2)(1-r_3)(1-r_4)|$, so all we need to do is plug $z = 1$ into $|(z-r_1)(z-r_2)(z-r_3)(z-r_4)| = |z^4 + z^3 + z^2 + z + 1|$. This yields $1 + 1 + 1 + 1 + 1 = 5$. Recall, we need to multiply this by 625, so the final answer is $5 * 625 = 3125$
6. B First, $\frac{1+i\sqrt{3}}{\sqrt{3}-i} = \frac{2\,cis\left(\frac{\pi}{3}\right)}{2\,cis\left(-\frac{\pi}{6}\right)} = cis\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = cis\left(\frac{\pi}{2}\right) = i$. Motivated by the approximate 2:1 ratio between the exponents of $5 + 12i$ and $-119 + 120i$, we try computing $(5+12i)^2$. $(5+12i)^2 = 25 - 144 + 120i = -119 + 120i$. So, $(5+12i)^{31} = (5+12i) * (5+12i)^{30} = (5+12i)(-119+120i)^{15}$. $\frac{(5+12i)^{31}}{(-119+120i)^{15}} = (5+12i)$. Our answer is $(5+12i) * i = -12 + 5i$
7. B Note that the angle is equal to $\arctan\left(\frac{1}{100}\right)$. Using small-angle approximation, $\tan(\theta) \sim \theta$ when θ is small. Thus, $\arctan(\theta) = \theta$ when θ is small.
*Note that $\theta \sim 0.009999$
8. B In this situation, Team A will have a team score of $7 * 4 + 2 * 10i = 28 + 20i$, while Team B will have a team score of $3 * 4 + 3 * 10i = 12 + 30i$. The magnitude of the score of Team A is $\sqrt{20^2 + 28^2} = \sqrt{400 + 784} = \sqrt{1184}$. The magnitude of the score of Team B is $\sqrt{12^2 + 30^2} = \sqrt{144 + 900} = \sqrt{1044}$. $1184 > 1044$, so Team A is the winning team. $\sqrt{1184} = 4\sqrt{74}$
9. C For this problem, we will consider the minimum score Team A could have gotten given how many tossups they got correctly, and the maximum score Team B could have gotten given how many tossups Team A got correctly. We can calculate this with the assumptions that 1) Team A gets no bonuses correct, and 2) Team B gets all

the remaining toss-ups correct and all the associated bonuses. We can make a table from keeping track of these scores if Team A gets anywhere from 5 to 10 tossups correct

Team A Tossups	Team A Minimum Score	Team B Maximum Score
5	20	20 + 50i
6	24	16 + 40i
7	28	12 + 30i
8	32	8 + 20i
9	36	4 + 10i
10	40	0

Examining the table, Team B has a score with magnitude well above Team A's score for 5, 6, and 7 toss-up questions answered correctly by Team A. At 8 toss-ups, Team A's score magnitude becomes larger than Team B's for the first time.

10. D Since the final complex scores need to be equal, the real and imaginary components need to be equal. This means they get the same number of toss-ups and bonuses. Let
$a =$ the number of tossups each team get
$b =$ the number of bonus questions each team get
We get the inequality $0 \leq b \leq a \leq 5$ since the maximum number of toss ups both team can get is 5 and they cannot get more bonus questions than tossups.
Solving the number of $b + (a - b) + (5 - a) = 5$ is simply stars and bars

$$\binom{5+3-1}{3-1} = 21$$

11. D The graph of $|z| + |1 + z| = 2$ is an ellipse with $2a = 2 \rightarrow a = 1$, and $2c = 1 \rightarrow c = \frac{1}{2}$. $b = \sqrt{a^2 - c^2} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$. The area enclosed by the ellipse is given by $\pi ab = 1 * \frac{\sqrt{3}}{2} * \pi = \frac{\pi\sqrt{3}}{2}$

12. D Since 8 of the 9 points remain the same, it is convenient to compute the average of these 8 points first; it will remain the same. From $z^8 - 1 = 0$, the 8th roots of unity sum to $-\frac{b}{a} = 0$, so the average of these 8 points is 0. So, assuming (x_9, y_9) are the coordinates of the point with magnitude 2 (starts out at $2 + 0i$), $x_1 + x_2 + x_3 + \ldots + x_8 = 0$ and $y_1 + y_2 + y_3 + \ldots + y_8 = 0$. This means the coordinates of the average point are $\left(\frac{x_9}{9}, \frac{y_9}{9}\right)$. This is just a scaled down version of the circle traced by the point with magnitude 2. Specifically, each dimension is scaled down by a factor of 9, so the area is $2^2 * \frac{\pi}{9^2} = \frac{4\pi}{81}$

13. B $f(x) = 2e^{7ix}$ satisfies everything given. Thus $f\left(\frac{11\pi}{3}\right) = 2e^{\frac{77\pi}{3}i} = 1 - i\sqrt{3}$

*Note that $2e^{ax}$ is the general form of the functional equation

$$2f(x + y) = f(x)f(y)$$

if differentiability is given.

There is technically an infinite number of possible values for a that satisfy the initial condition, so long $\frac{a\pi}{6}$ is coterminal to $\frac{7\pi}{6}$. In other words, $a = 12k + 7$ for any integer k. They all yield the same outcome, as $12k\left(\frac{11\pi}{3}\right)$ is a multiple of 2π.

14. E We want the smallest possible value of $|r| = |\theta|$, where θ is coterminal with $\frac{\pi}{3}$. Since θ is in the domain of all real numbers, it can be positive or negative, the 2 values to consider are $\theta = \frac{\pi}{3} + 2\pi$, and $\theta = \frac{\pi}{3} - 2\pi$. The magnitudes are, respectively, $\frac{7\pi}{3}$ and $\frac{5\pi}{3}$. The lesser of these 2 values is $\frac{5\pi}{3}$

15. A The numerator can be factored over complex numbers as $\sqrt{(x+i)(x-i)}$, so the limit becomes $\lim_{x\to i}\frac{\sqrt{(x+i)(x-i)}}{x-i} = \sqrt{\frac{x+i}{x-i}}$. If we plug in $x = i$, we get $\sqrt{\frac{2i}{0}}$. There is a 0 in the denominator but not in the numerator, so this limit does not exist

16. D There is not enough information to solve for a, b, c, and d directly. We need to figure out a way to find $9b - 16c$ despite this. $x + yi = a + bi + ci - d = (a - d) + (b + c)i$, so we know $Re(x + yi) = a - d = 1$. Also, $3x + 4y = 5 + 10i \to 3a + 3bi + 4c + 4di = 5 + 10i$. Setting the real and imaginary parts equal individually, $3a + 4c = 5$, and $3b + 4d = 10$. Since there is only one equation with b, we scale it in order to have a $9b$: $(3b + 4d) * 3 = 9b + 12d = 10 * 3 = 30$. From $a - d = 1$, we have $12d = 12 * (a - 1) = 12a - 12$. From $3a + 4c = 5$, we have $12a = 4 * (5 - 4c) = 20 - 16c$. Finally, $9b + 12d = 9b + (12a - 12) = 9b + (20 - 16c) - 12 = 30 \to 9b - 16c = 30 + 12 - 20 = 22$.

17. A Let θ be the answer. The magnitude of the complex number $20 + 21i$ is $\sqrt{20^2 + 21^2} = \sqrt{400 + 441} = \sqrt{841} = 29$. This means $\cos(\theta) = \frac{20}{29}, \sin(\theta) = \frac{21}{29}. \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{21}{20}$. The reciprocal functions will by 1 divided by the respective normal function. The only choice that represents one of these relationships accurately is A.

18. C It is possible to square both sides of the equation and solve for a and b directly. However, notice that the right-hand side can be written as $\sqrt{(5-1) + 2\sqrt{-1}\sqrt{5}} = \sqrt{\left(\sqrt{5}^2 + i^2\right) + 2i\sqrt{5}} = \sqrt{\left(\sqrt{5} + i\right)^2}$. So, $z = \sqrt{5} + i$, and $a + b\sqrt{5} = \sqrt{5} + 1 * \sqrt{5} = 2\sqrt{5}$

19. D $\ln\left(1 - i\sqrt{3}\right) = \ln\left(2\, cis\left(\frac{-\pi}{3}\right)\right) = \ln(2) + \ln\left(e^{-\frac{i\pi}{3}}\right) = \ln(2) - \frac{i\pi}{3}$.

20. E $cis(a) * cis(b) = cis(ab)$, so $\prod_{n=-90}^{90} cis(n°) = cis(\sum_{n=-90}^{90} n°) = cis(0) = 1$

21. A By inspection, the two positive roots of $2^x - x^2$ are 2 and 4 ($2^2 = 2^2, 2^4 = 4^2$). The distance between 2 and 4 on the Argand Diagram is $4 - 2 = 2$

22. B $2^a - a^2 = 0 \to 2^a = a^2$. $\ln(2^a) = \ln(a^2) \to a\ln(2) = 2\ln(a)$. $a = r\, cis(\theta)$, so $r\, cis(\theta) * \ln(2) = 2\ln\left(r\, cis(\theta)\right) = 2(\ln(r) + i\theta)$. $cis\,\theta = \frac{2\ln(r) + 2i\theta}{r\ln(2)}$. $\sin(\theta) = Im\left(cis(\theta)\right) = \frac{2\theta}{r\ln(2)}$.

*Note that there are non-real solutions, thus D is not an acceptable answer choice.

23. E A: $2^x \gg x^2$ as $x \to \infty$

B: $2^x - x^2 = -x^2$ as $x \to -\infty$
C: All numbers are complex, thus 2 is a complex root.
D: Graphing $2^x, x^2$ quickly reveals it does have a negative real root.

24. C $x = 1$ is not a non-real solution, so we want the number of non-real solutions to $x^3 - 4x^2 + 10x - 20 = 0$. Since the coefficients are real, we know this equation will have an even number of non-real solutions. The sum of squares of the roots (with roots a, b, c) is $a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + ac + bc) = \left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right) = (-4)^2 - 2(10) = 16 - 20 = -4. -4 < 0$, so there is at least 1 non-real solution. This means there are 2 non-real solutions.

25. A Let $a = 1 + \frac{x}{1+\frac{x}{\dots}}$. Then $x = \frac{4}{a}$ from the original equation. Also, $a = 1 + \frac{x}{a} \to a = 1 + \frac{\left(\frac{4}{a}\right)}{a}. a^3 - a^2 - 4 = 0. a = 2$ works. x is real so $a = \frac{4}{x}$ is real, and $a = 2$ is the only real root. $x = \frac{4}{a} = \frac{4}{2} = 2$

26. E I is false: This property does not hold for complex numbers. For example, $\sqrt{-1} * \sqrt{-1} = i * i = -1 \neq \sqrt{(-1)(-1)} = 1$. II,III are also false: For example, the principal value of $\ln((-1)^2) = \ln(1) = 0 \neq 2\ln(-1) = 2\pi i$.

27. A The 2nd and 3rd equations add up to the requested quantity $((7x + 4y + 5z) + (5x + 6y + 7z) = (12x + 10y + 12z))$. So, $(6 + i) + (4 + i) = 10 + 2i$

28. D Note that it does not work to assume the roots of $f(x)$ come in complex conjugates, because that would require all the coefficients to be real, which is not the case.
Let $g(x) = f(xi)$

$$g(x) = x^4 - aix^3 - bx^2 + cix + d$$

By definition, $g(x)$ has real coefficients, thus having complex conjugate roots. Since $\frac{(2+i)}{i} = 1 - 2i$, $\frac{(1+3i)}{i} = 3 - i$ are roots of $g(x)$,
$(1 + 2i)i = i - 2, (3 + i)i = -1 + 3i$ must be roots of $f(x)$

$$-\frac{b}{a} = (2 + i) + (-2 + i) + (1 + 3i) + (-1 + 3i) = 8i$$

29. B A is a line (the perpendicular bisector).
C is the null set because it is not possible for the difference of distances to two points to be greater than the distance between the points.
D is the entire Argand plane.

30. A Adding the two equation resembles $cis(x)$. Multiplying the second equation by i then adding gives

$$cis(x) + \frac{cis(2x)}{2} + \frac{cis(3x)}{4} \dots = \frac{1}{2} + ri$$

Using the fact that $e^{ix} = cis(x)$, the expression is geometric series with first term e^{ix} and common ratio $\frac{e^{ix}}{2}$

$$\frac{e^{ix}}{1 - \frac{e^{ix}}{2}} = \frac{1}{2} + ri$$

$$\rightarrow \frac{1-\frac{e^{ix}}{2}}{e^{ix}} = \frac{\frac{1}{2}-ri}{r^2+\frac{1}{4}}$$

Adding $\frac{1}{2}$ to both sides,

$$e^{-ix} = \frac{\left(\frac{1}{2}+\frac{r^2}{2}+\frac{1}{8}\right)-ri}{r^2+\frac{1}{4}} = \frac{(4r^2+5)-8ri}{8r^2+2}$$

Using the fact that the magnitude of $e^{-ix} = 1$,

$$(8r^2+2)^2 = (4r^2+5)^2 + (8r)^2$$
$$64r^4+32r^2+4 = 16r^4+40r^2+25+64r^2$$
$$48r^4-72r^2-21=0$$
$$16r^4-24r^2-7 = (4r^2-7)(4r^2+1) \rightarrow r = \frac{\sqrt{7}}{2}$$